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ROTATION SAMPLING: INTRODUCTION AND REVIEW OF RECENT DEVELOPMENTS

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ABSTRACT

Rotation sampling has emerged as one of the important sampling strategies from its beginning in the mid twentieth century and has widely been applied in several welfare schemes related to the human resource developments. This paper gives a brief review of development of the existing work in the literature in chronological order. The problems of estimation of population mean and use of auxiliary information in rotation sampling have been discussed in detail through some key papers. The main purpose of the paper is to show how important methodologies, or mathematical tools have helped to develop the theory. Without any claim of completeness, a modest attempt has been made to crystallize many of the basic ideas available in the literature.

Key words: Rotation Sampling, Auxiliary Information, Chaining of estimators

INTRODUCTION

Sample surveys now a days are not limited to one-time enquiries. In survey sampling, the sampler is interested not only in estimating the value of the character for the most recent occasion but also estimating the change in the value of the character from one occasion to the next. This interest is reflected most strikingly in the periodic census of population, housing, manufacturing etc. that are conducted in many countries. Over time some units may fall out from the population and new units may enter. If the aim is to estimate changes taking place in the characteristic over time or to estimate average over a certain period of time, one-point sample surveys do not serve the purpose. In order to meet the specific objective, surveys are often repeated over several occasions. A repeated survey conducted after a given period of time or if possible, at regular intervals, provides not only continuity of data but also improve estimates of the population characteristics by taking into account the information already available from past surveys and of the changes taking place in it. For example, in many countries, labor-force surveys are conducted monthly to estimate the rate of unemployment. In Indian agricultural survey one may be interested (i) in estimating the average yield per acre of an important crop (say wheat) in current season, (ii) estimating the change in average yield for a province (county) for two different seasons, and (iii) estimating both parameters from (i) and (ii) simultaneously. Other examples may be monthly surveys in which data on price of goods are collected to determine a consumer price index, and political opinion surveys conducted at regular intervals to measure preferences of voters.

An important aspect of successive sampling (continuous survey) is the structure of the sample on each occasion. The structures of the sample must be guided by the following three different types of requirement in the successive sampling:

(a) A new sample on each occasion (Repeated Sampling)
(b) Same sample observed on each occasion (Panel Sampling)
(c) On the $(i+1)^{th}$ occasion, choose a sub-sample of the sample observed on the i-th occasion (Sampling on Successive Occasions or Rotation Sampling). A detail for Repeated, Panel and Rotation Sampling techniques are mentioned in Singh (2003)

Repeated Sampling

In repeated sampling where the basic objective is to study the overall average or total, it is better to select a fresh sample on each occasion.

Panel Sampling

The main objective of panel sampling is to estimate the change with a view to study effect of the forces acting upon the population. For this, it is better to retain the same sample from occasion to occasion.

Rotation Sampling

In rotation sampling the objective is to estimate the average value for the most recent (current) occasion, the retention of a part of the sample over occasions provides efficient estimates as compared to other alternatives (Singh and Choudhary 1986). In most of the surveys interest centers on the rotation sampling, particularly, if the characteristic of the population are likely to change rapidly with time. Hence, replacement of the part of the sample on each occasion leads to a better result. This procedure of partial replacement of units is designated by different names in survey literatures. Researchers call this procedure as "Sampling on Successive Occasions with Partial Replacement of Units" or "Rotation Sampling" or "Rotation Designs for Sampling on Repeated Occasions" or "Sampling for Time Series".

The objective of this paper is to provide a review of various methods of rotation sampling available in literature in chronological manner. Considering the complexity the attention is restricted to the estimation of population mean in two occasion rotation patterns. However a good amount of work have been done by different authors in the area of estimation of other population parameters such as median, total, ratio, quantiles etc.

ESTIMATION OF POPULATION MEAN IN ROTATION SAMPLING

The Beginning

The theory of rotation (successive) sampling was initiated with the work of Jessen (1942). He propounded the idea of partial replacement of units drawn on the first occasion for getting an improved estimator of the population mean on the second occasion. In fact, he combined two independent estimators of the population mean on the second occasion – a double sampling regression estimator based on the units common to both the occasions and a sample mean of new (fresh) units, to provide an improved estimator of the population mean on the second occasion. In the following sections discussion will be made for the estimation of population mean on current occasion in two occasion rotation patterns with the aid of some revolutionary works.

Notations

Let $U = (U_1, U_2, \ldots, U_N)$ be the finite population of N units, which has been sampled over two occasions. The character under study is denoted by x (y) on the first (second) occasion respectively. Let a simple random sample (without replacement) of size n be selected on the first occasion. A random sub-sample of $m = n\lambda$ units is retained (matched) for its use on the second occasion, while a fresh simple random sample (without replacement) of $u = (n\text{-}m) = n\mu$ units is drawn on the second occasion from the entire population so that the sample size on the second occasion is also n. λ and μ $(\lambda + \mu = 1)$ are the fractions of matched and fresh samples at the second (current) occasion. Let us denote this sampling strategy as scheme A. Further, the following notations have been considered throughout this work:

$\bar{X}, \bar{Y}, \bar{Z}$: Population means of the variables x, y and z respectively.

$\bar{x}_n, \bar{x}_m, \bar{y}_u, \bar{y}_m, \bar{z}_u, \bar{z}_n, \bar{z}_m$: Sample means of the respective variables based on the sample sizes shown in suffices.

$\rho_{yx}, \rho_{yz}, \rho_{xz}$: Correlation coefficients between the variables shown in suffices.

$S_X^2 = (N-1)^{-1} \sum_{i=1}^{N} (x_i - \bar{X})^2$: Population mean square of the variable x.

S_Y^2, S_Z^2 : Population mean squares of the variables y and z respectively.

β_{yx} : Population regression coefficient.

C_2 : Cost of the survey on second occasion.

C_u, c_m : Cost of the survey for the sample sizes shown in suffices.

Jessen's Approach (1942)

For estimating population mean $\bar{Y}$ at current occasion based on successive sampling scheme A under the above notations, two independent estimators could be defined (Jessen 1942, Cochran 1963) as:

(i) $\bar{y}_u$ based on sample of size u, drawn afresh on current occasion

(ii) $\bar{y}'_m = \bar{y}_m + \beta_{yx}(\bar{x}_m - \bar{x}_n)$ is the linear regression estimate of $\bar{Y}$ based on sample of size m, common to both the occasions.

Final estimator is the convex linear combination of these two estimators and is given by

$$\hat{\bar{Y}} = \varphi\bar{y}_u + (1-\varphi)\bar{y}'_m \qquad (1)$$

where φ (0 d" φ d" 1) is unknown constant to be determined under certain criterion.

The variance of the estimator $\hat{\bar{Y}}$ is $V\left(\hat{\bar{Y}}\right) = \left(\frac{1-\mu\rho_{yx}^2}{1-\mu^2\rho_{yx}^2}\right)\frac{S_y^2}{n}$

(Cochran, 2007) (2)

For the case of complete matching (u=0) or no matching (m = 0), $V\left(\hat{\bar{Y}}\right) = S_y^2/n$.

Minimizing (2) w.r.t μ, the optimum values of μ and λ are obtained as

$$\mu_{opt} = \frac{1}{1+\sqrt{1-\rho_{yx}^2}}, \quad \lambda_{opt} = \frac{\sqrt{1-\rho_{yx}^2}}{1+\sqrt{1-\rho_{yx}^2}} \qquad (3)$$

And the corresponding optimum value of variance is

given by $V\left(\hat{\bar{Y}}\right)_{opt} = \frac{\left(1+\sqrt{1-\rho_{yx}^2}\right)}{2n}$ (4)

Efficiency of successive (rotation) sampling estimator $\hat{\bar{Y}}$ w.r.t sample mean

estimator $\bar{y}_u$ is $E = \frac{2}{1+\sqrt{1-\rho_{yx}^2}}$ (5)

From equation (3) it is evident, for ρ = 1 and 0, λ_{opt} is 0 and 50 respectively, which explains that the optimum percentage to be obtained for matching decreases with increasing value of ρ. However, for $m=0$, β_{yx} cannot be obtained and the results derived above become invalid. For $\rho=1$ therefore, m should be taken as 2 which will completely determine the regression equation. For $\rho = 0.5$ and 0.7, optimum value of λ are 0.46 and 0.42 respectively. Thus, in practice more than 50% of the sample units will never have to be matched. The percentage gain in efficiency (E-1) 100% increases with increasing value of ρ (for optimum matching %). The percent gain is moderate (< 25%) unless ρ exceeds 0.8. If one uses a predetermined value of λ, then also the percentage gain in efficiency increases with increasing value of ρ. For $\lambda=1/3$, maximum percentage gain is 67% for $\rho = 1$. When N is infinite, Patterson (1950) showed that the linear unbiased estimator of $\bar{Y}$ is given by (1). Tikkiwal (1955) showed that $\hat{\bar{Y}}$ is best unbiased linear estimator with variance (2), if N is finite.

Yates Approach (1949)

Yates extended the work of Jessen to develop the theory for more than two occasions where the population mean of the character is estimated on each of h>2 occasions. Subsequently, Narain (1953) and Tikkiwal (1950, 1951, 1953, 1956, 1958, 1964) published a series of interesting papers in connection with problem in question under same set up of estimation as given by Jessen (1942). Tikkiwal (1956) studied the case when estimated values of population regression coefficient and correlation coefficient are taken into account in minimum variance linear unbiased estimator (MVLUE) under the assumption of normality of population. Further Tikkiwal (1960) showed that the estimator satisfied the unbiasedness in extended sense. Following these works, Sisodia (1985) studied the effect on MVLUE of a population mean on the second occasion in successive (rotation) sampling when sample values of regression and correlation coefficients are used in it.

Kulldorff Approach (1963)

Kulldorff considered the problem of optimum allocation for simple random sampling on two-occasion at length by taking into account the cost of the survey. He considered the case where on the second occasion the cost of measuring a matched unit may differ from that of measuring a new unmatched unit and does not assume equal sample sizes on two occasions. Thus, apart from fixed costs, cost on the second occasion is $\frac{C_2}{c_u} = m\delta + u$ where $\delta = c_m / c_u$. If sample sizes are the same on the two occasions $m + u = n$, the optimum unmatched proportion on the second occasion is obtained by minimizing

$$\frac{V(\bar{y}_u)C_2}{c_u S_y^2} = \left[\delta+\mu(1-\delta)\right]\frac{\left(1-\mu\rho_{yx}^2\right)}{\left(1-\mu^2\rho_{yx}^2\right)} \qquad (6)$$

He also discussed the case where the costs are to be same on the two occasions.

Pathak and Rao Approach (1967)

Pathak and Rao proposed some more efficient estimators under the above schemes of sampling over two-occasion. They showed that

$$Y^* = N\varphi\left[\frac{m\bar{y}_u + \lambda\bar{y}_m}{n-m}\right] + N(1-\varphi)\bar{y}'_m \qquad (7)$$

is more efficient in having the smaller exact variance than Cochran (1963) for infinite population. Ghangurde and Rao (1969) showed that Kulldorff's estimator has smaller large sample minimum variance than that of Pathak and Rao (1967).

Patterson Approach (1950)

Following Yates (1949), Patterson (1950) studied the problem through a more general approach relaxing assumptions on constancy of replacement fraction and variances. He considered the minimum variance linear unbiased estimator of $\bar{Y}$ under sampling scheme A. A best linear (linear in terms of observed means) unbiased estimator (BLUE) of $\bar{y}$ was introduced as

$$\hat{\bar{Y}}^* = a_1\bar{x}_m + a_2\bar{x}_n + a_3\bar{y}_m + a_4\bar{y}_u \qquad (8)$$

where, the constants a_i $(i = 1, 2, 3, 4)$ and the matching fraction λ are to be suitably determined so as to minimize the variance. $S_x^2 = S_y^2$. Taking expectations (8) gives

$$E(\hat{\bar{Y}}^*) = (a_1+a_2)\bar{X}+(a_3+a_4)\bar{Y}$$

Now unbiasedness requires $a_1+a_2 = 0,\ a_3+a_4 = 1$.

Hence, $\hat{\bar{Y}}^* = a_1\left(\bar{y}_m - \bar{x}_n\right) + a_3\bar{x}_m + (1-a_3)\bar{y}_u$

The optimum value of the matching fraction is $\lambda^* = \dfrac{\sqrt{1-\rho_{yx}^2}}{(1+\sqrt{1-\rho_{yx}^2})}$ (9)

The corresponding value of variance of $\hat{\bar{Y}}^*$ is $V_{opt}(\hat{\bar{Y}}^*) = \dfrac{S_y^2(1+\sqrt{1-\rho_{yx}^2})}{2n}$ (10)

The results (9) and (10) coincide with the results of Jessen (1942), showing that the best linear combination of the regression estimator for the matched sample and the mean per unit estimator for the unmatched sample is in fact the best linear unbiased estimator for the current population mean in SRS on two occasions (Cochran 2007).

Ekler Approach (1955)

Ekler extended this method to two-level and three-level rotation sampling. He compared three methods of rotation sampling on cost basis. He also extended Cochran's work in determining optimum patterns for one-level rotation sampling estimate. Further Rao and Graham (1964), Singh and Singh (1965), Raj (1965), Abraham et al. (1969), studied the problem of partial replacement of sample units according to some specific patterns. Alternative estimators, called composite estimators, which are generally less efficient than the MVLUE, were obtained by Hansen et al. (1955).The correlation model, i.e., correlations between character on i^{th} and j^{th} occasions, plays an important role in successive sampling on h-occasions.

The variance of MVLUE considered by Patterson (1950) and Tikkiwal (1951) under general correlation model was derived by Ajgaonkar (1968).

Avadhani and Sukhatme Approach (1970)

Avadhani and Sukhatme investigated the problem of rotation sampling on two occasions by comparing the efficiencies of the sampling procedure proposed by Rao, Hartley and Cochran (1962) and the ratio method of estimation for SRSWOR under a finite population model. Sen et al. (1975) used the information on the study variable from the previous occasion through ratio method of estimation in successive (rotation) sampling. Gupta (1970) has suggested the use of product method of estimation in successive (rotation) sampling and later Artes et al. (1998), Artes and Garcia (2000, 2001), extended the methodology. Arnab (1998) used the data collected on the first occasion for selection of sample as well as stratification for the second occasion. The problem of matching primary and secondary stage units in multi-stage sampling on successive occasions was first examined by Kathuria (1959) and Singh and Kathuria (1969). Different types of replacement policies of sample in multi-stage sampling on successive occasions were further discussed by Kathuria (1975), Kathuria and Singh (1971 a), Singh (1970) and Srivastav and Singh (1974).

UNEQUAL PROBABILITY SAMPLING SCHEMES FOR ESTIMATING CURRENT POPULATION MEAN

The concept of varying probability sampling scheme in rotation (successive) sampling was at first explored by Raj (1965).

Raj's Approach (1965)

A sample under scheme A is selected by ppswr from U using q as a size-variable ($p_i = q_{i/}Q$, size measure of i). An independent sample of size u is selected from U by ppswr such that $(m + u = n)$. The estimator is

$$\hat{Y}_R = \varphi t_m + (1-\varphi) t_u,\ 0<\varphi<1, \tag{11}$$

$$t_m = \sum_{j\in m} \frac{(y_{mj} - x_{mj})}{mp_j} + \sum_{j\in n} \frac{x_{nj}}{np_j},\ t_u = \sum_{j\in u} \frac{y_{uj}}{up_j}$$

where t_u and t_m are the estimators based on the sample of sizes u and m respectively.

Assuming $\sum_{j=1}^{N} P_j(\frac{X_j}{P_j}-X)^2=\sum_{J=1}^{N} P_j(\frac{Y_j}{P_j}-Y)^2=V_{0pt}$ for the optimum sampling fraction

$$\lambda = m/n, V(\hat{Y}) = \frac{V_0(1+\sqrt{2(1-\delta)})}{2n} = V_R \qquad (12)$$

For $\delta>1/2$, V_R is less than the variance of the estimator $\hat{Y}_{pps}$ under sampling with no matching.

Ghangurde and Rao's Approach (1969)

As per this approach a sample of size n is selected from U by the Rao-Hartley-Cochran (1962) sampling procedure using $p_i = q_{i/}Q$. An independent sample of size u is drawn from U by using the RHC- procedure, using the same p_i values.

The estimator is given by $\hat{Y}_{GR} = \varphi t_m + (1-\varphi)t_u, 0 \in \varphi \in 1$ (13)

$$t_m = \sum_{j\in m}\frac{(y_{mj}-x_{mj})\pi_j n}{mp_j}+\sum_{j\in n}\frac{x_n\pi_j}{p_j},\ t_u = \sum_{j\in u}\frac{y_{uj}\pi_j^*}{p_j}$$

where t_u and t_m are the estimators based on the sample of sizes u and m respectively. π_j, π_j^* denote the sums of p values included in the groups of unit j while selecting sample of sizes n and u respectively by the RHC method.

Adopting the generalized least square approach of Gurney and Daly (1965) to survey sampling Singh (1968), Chaudhuri and Arnab (1979) suggested simple alternative improved estimators for sampling procedures. Raj (1965) techniques were further investigated by Avadhani and Srivastava (1972) and Kathuria (1973). The use of double sampling for stratification in repeat surveys was made by Singh and Singh (1965). The study of multiple characters on successive occasions was examined by Singh and Singh (1973).

Another development in this area is the estimation of population ratio of two characters on the second occasion. Let x and y be two character with notations $\bar{X}_1, \bar{X}_2$, $x_{1j}, x_{2j}, \bar{x}_1^{'}$ (sample mean of the variable at first occasion), $\bar{x}_1^{''}$ (sample mean of the variable at second occasion), $\bar{x}_2^{'}, \bar{x}_2^{''}, R_1 = Y_1/X_1, r_1 = \bar{y}_1/\bar{x}_1, r_2 = \bar{y}_2/\bar{x}_2$ having obvious meanings.

Rao (1957), and Rao and Pereira Approach (1968)

This approach (Rao 1957, Rao and Pereira 1968) considered the same sample selected by the srswor on both occasions ($\lambda = 0$). Rao proposed the ratio type

estimator $\hat{R}_2^{(1)} = \frac{r_2 R_1}{r_1}$ (14)

Rao and Pereira considered the product type estimator $\hat{R}_2^{(2)} = \frac{r_2 r_1}{R_1}$ (15)

Both the estimators assume, R_1 to be known. Tripathi and Sinha (1976) considered the sampling scheme A. They proposed the estimator

$$\hat{R}_2^{(3)} = \varphi\, \hat{R}_{2m} + (1-\varphi)\, \hat{R}_{2u},\ 0 \le \varphi \le 1 \qquad (16)$$

where $\hat{R}_{2m} = \frac{\left\{\bar{y}_2^{'} + b_y\left(\bar{y}_1 - \bar{y}_1^{'}\right)\right\}}{\left\{\bar{x}_2^{'} + b_x\left(\bar{x}_1 - \bar{x}_1^{'}\right)\right\}}$ and $\hat{R}_{2u} = \frac{\bar{y}_2^{''}}{\bar{x}_2^{''}}$ are based on the sample of sizes m and u at second occasion. b_y ,b_x, are respectively the regression coefficient of y_2 on y_1 and of x_2 on x_1, φ is suitable constant. Further development in the area of sampling on two occasions are due to Adhvaryu (1978),Chaturvedi and Tripathi (1983), Chaudhari (1985), among others.

USE OF AUXILIARY INFORMATION IN ROTATION SAMPLING

In sample surveys, sometimes, it is possible to measure certain characters other than the study character which are highly correlated with the study character. The additional information, thus obtained, is known as the ancillary or auxiliary or apriori information and the characters on which this information is obtained, are known as ancillary or auxiliary characters. In most of the survey situations, auxiliary information is either available or may be made available in one form or the other by diverting a part of the resource available for this purpose.

The use of auxiliary information in probability sampling has been a commonly used device to improve the precision of estimates of certain population parameters obtained from a probability sample. If used intelligibly, this information provides us with the sampling strategies better than those in which no such auxiliary information is used. It is a well-known fact that in whatever form the auxiliary information is available, one can always devise suitable ways or procedures of using it in obtaining the sampling strategies, which are more precise. Beside its advantages, the main drawback of the use of auxiliary information is that such estimates usually become biased.

The use of auxiliary information in sample surveys may be done in three basic ways - (a) at pre selection stage (planning) or designing stage, (b) at selection stage and (c) at estimation stage. Auxiliary information may also be used in mixed ways; for example, information on a character w may be used in defining a set of inclusion probabilities and that on another character z may be used in constructing some efficient estimators for certain population parameters.

In most of the survey situations where auxiliary information is available in one form or the other, the objective of the survey statistician becomes to choose such sampling strategies and estimation procedures so as to give the most efficient estimator for population characteristics; usually population mean, population total, population variance of the study variable or ratio of two population characteristics etc.

In whatever form the auxiliary information is available, it is always possible to devise suitable ways of using it in obtaining the sampling strategies which are better than those in which no such information is used. Two such strategies are known as "ratio method of estimation" and "regression method of estimation". Another well-known sampling strategy which utilizes the apriori information is "product method of estimation". Contrary to the ratio method of estimation, which is used generally when the correlation coefficient between y and z is positive (that is, $\rho_{yz} > 0$), the product method of estimation is applicable when $\rho_{yz} < 0$. It is obvious that all these sampling strategies require knowledge of $\bar{Z}$.

In some situations of practical importance, information on more than one auxiliary character correlated with the study variable are available. In such cases, multivariable ratio or regression-type estimators are used.

Sen's Approach (1971)

Utilizing the availability of two auxiliary variable x and z with unknown mean, Sen (1971) suggested multivariate ratio estimate from matched portion and simple mean form the fresh portion in two occasion rotation sampling. In particular he considered x to be the value of y on first occasion. Following scheme A he proposed

$$\bar{y}'_R = \varphi\, \bar{y}_R + (1-\varphi)\bar{y}_u \qquad (17)$$

where $\bar{y}_R = \varphi_1 \dfrac{\bar{y}_m}{\bar{x}_m}\bar{x}_m + (1-\varphi_1)\dfrac{\bar{y}_m}{\bar{z}_m}\bar{z}_m$, φ_1 is constant to be determined to maximize the precision of $\bar{y}_R$. Raj (1965) had earlier generalized this methodology for k($\geq$ 1) auxiliary variables for the case when difference estimator is used from the matched portion. Sen (1972, 1973) extended the theory to provide optimum estimate in sampling on two occasion by using a double sampling multivariate ratio estimate using k ($\geq$1) auxiliary variables from the matched portion of the sample. Shah and Patel (1985) considered the problem of successive sampling over two-occasions when one or more auxiliary information is assumed to be known at the first occasion. Singh (1990), Singh et al. (1991) and Singh and Singh (2001) proposed estimators for current mean in successive sampling when information on an auxiliary variable is available at the current occasion. Singh (2003) extended their work for h (>2) occasion successive sampling. Tankou and Dharmadhikari (1989) considered some improved ratio-type estimators in successive sampling.

Feng and Zou's Approach (1997)

Feng and Zou (1997) used the information on auxiliary variable for all the units of the finite population and suggested the estimator of the population mean as

$$\hat{\bar{Y}} = \varphi \frac{\bar{y}_{u-m}}{\bar{z}_{u-m}} + (1-\varphi)\left[\frac{\bar{y}_m}{\bar{z}_m} + B_{21}\left(\frac{\bar{x}_n}{\bar{z}_n} - \frac{\bar{x}_m}{\bar{z}_m}\right)\right]; \; n \neq u+m \qquad (18)$$

where $$B_{21} = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \frac{\bar{Y}}{\bar{Z}} Z_i\right)\left(X_i - \frac{\bar{X}}{\bar{Z}} Z_i\right) / \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \frac{\bar{X}}{\bar{Z}} Z_i\right)^2$$

They defined a necessary and sufficient condition for $\hat{\bar{Y}}$ to be more efficient when no auxiliary variable is used.

Biradar and Singh (2001) proposed an estimator more efficient than the sample mean and the estimators considered by Sen (1971), utilizing information available on both the occasions on an auxiliary variate with unknown population mean. Feng and Zou (2001), Singh (2005), Singh and Priyanka (2006, 2007, 2008), Singh and Viswakarma (2007, 2009), Singh and Karna (2009 b) developed sampling strategies in successive sampling using auxiliary information on both fresh and matched portion of the sample collected on the second occasion.

CHAINING OF THE ESTIMATORS IN ROTATION SAMPLING

On the other hand, sometimes information available on auxiliary variables is used in "chaining the estimators". For example, in several situations the information on $\bar{Z}$, the mean of the ancillary character z highly correlated with study character y is not known, rather, information on another auxiliary character w is completely known which is closely related to z but compared to z remotely related to y. It is then advisable to obtain an estimate of $\bar{Z}$ from a preliminary sample of size n' $(n' > n)$ with the help of auxiliary character w and then to use the estimated value of $\bar{Z}$ in place of $\bar{Z}$ in forms of ratio, regression or product method of estimation. Such estimators are termed as "chain-type estimators". There are various ways for utilizing the available auxiliary information at estimation stage in successive (rotation) sampling. Chaining of auxiliary information in two-phase structure is one of them, viz Chand (1975), Kiregyera (1980, 1984), Mukerjee et al. (1987), Singh and Singh (1991), Singh et al. (1994), Singh and Upadhyaya (1995), Singh (2001) among many others. Successive (rotation) sampling resembles two-phase sampling, hence, there is a greater scope to consider the chain-type estimators in successive sampling over different occasions. Motivated with these facts, Singh and Karna (2009 a), Singh and Prasad (2010), Singh et al. (2011a, b), Singh and Homa (2013), Singh et al. (2013), Singh et al. (2014), Bandyopadhyay and Singh (2015) carried out further study in this direction.

SAMPLING ON MORE THAN TWO OCCASIONS

The problem of estimating the current population mean for a univariate character was extended to h ($\geq$2) occasions by Yates (1949),Patterson (1950), Tikkiwal (1950), Kulldorff (1954), among others .When there are more than two occasions one has a large flexibility in using both sampling procedure and estimates of character. Thus on occasion i, one may have parts of the sample that are matched with occasions (i-1), parts that are matched with occasions (i-2), and so on. One may consider single multiple regressions of all previous matchings' on the current occasion. However, it has been seen that the loss of efficiency incurred by using the information from the latest two or three occasions only if fairly small in many circumstances (Kulldorff 1979). Since then several authors have suggested the methods of rotation sampling on more than two occasions, which have been reviewed by Cochran (2007). It is to be noted that all the methods discussed in the previous sections may be extended for more than two occasions.

CONCLUSION

This review serves as a brief introduction of the topic and its development over the period. The foundations of rotation sampling are based almost entirely on Jessen's (1942) fundamental paper. The review suggests that auxiliary information available in any form if used intelligibly, in the form of product, ratio, exponential or chain type estimators, may lead to enhancing the precision of estimates. Thus any research in this area will be beneficial if such information is utilized. Through this brief review it has been observed that the estimators proposed under rotation sampling are more efficient (lesser variance, higher percentage gain in efficiency) than the conventional sample mean estimators. Although the technique of rotation sampling has been recognized for many years; still the need of its application in the real life situations exits. In Indian perspective generally two-phase sampling is used in socio-economic, demographic and other surveys. Analyzing the novel features of the rotation sampling, it is suggested that statisticians and survey practitioners may apply this strategy in future.

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